APPENDIX

DE MORGAN'S ELEMENTS OF ARITHMETIC.

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I. ON THE MODE OF COMPUTING.

THE rules in the preceding work are given in the usual form, and the examples are worked in the usual manner.   But if the student really wish to become a ready computer, he should strictly follow the methods laid down in this Appendix ; and he may depend upon it that he will thereby save himself trouble in the end, as well as acquire habits of quick and accurate calculation.

I. In numeration learn to connect each primary decimal number, 10, 100, 1000, &c. not with the place in which the unit falls, but with the number of ciphers following.   Call ten a *one-cipher* number, a hundred a *two-cipher* number, a million a *six-cipher* number, and so on.   If *five* figures be cut off from a number, those that are left are hundred-thousands ; for 100,000 is a *five*-cipher number.   Learn to connect tens, hundreds, thousands, tens of thousands, hundreds of thousands, millions, &c. with 1, 2, 3, 4, 5, 6, &c. in the mind.   What is a *seventeen-cipher* number ?   For every 6 in seventeen say *million,* for the remaining 5 say *hundred-thousand :* the answer is a hundred thousand millions of millions.   If twelve places be cut off from the right of a number, what does the remaining number stand for ?—*Answer,* As many millions of millions as there are units in it when standing by itself.

II. After learning to count forwards and backwards with rapidity, as in 1, 2, 3, 4, &c. or 30, 29, 28, 27, &c., learn to count forwards or backwards by twos, threes, &c. up to nines at least, beginning from any number.   Thus, beginning from four and proceeding by sevens, we

have 4, 11, 18, 25, 32, &c., along which series you must learn to go as easily as along the series 1, 2, 3, 4, &c.; that is, as quick as you can pronounce the words. The act of addition must be made in the mind without assistance: you must not permit yourself to say, 4 and 7 are 11, 11 and 7 are 18, &c.; but only 4, 11, 18, &c. And it would be desirable, though not so necessary, that you should go back as readily as forward; by sevens for instance, from sixty, as in 60, 53, 46, 39, &c.

III. Seeing a number and another both of one figure, learn to catch instantly the number you must add to the smaller to get the greater. Seeing 3 and 8, learn by practice to think of 5 without the necessity of saying 3 *from* 8 *and there remains* 5. And if the second number be the less, as 8 and 3, learn also by practice how to pass *up* from 8 to the next number which ends with 3 (or 13), and to catch the necessary augmentation, *five*, without the necessity of formally undertaking in words to subtract 8 from 13. Take rows of numbers, such as

$$4 \quad 2 \quad 6 \quad 0 \quad 5 \quad 0 \quad 1 \quad 8 \quad 6 \quad 4$$

and practise this rule upon every figure and the next, not permitting yourself in this simple case ever to name the higher one. Thus, say 4 and 8 (4 first, 2 second, 4 from the next number that ends with 2, or 12, leaves 8), 2 and 4, 6 and 4, 0 and 5, 5 and 5, 0 and 1, 1 and 7, 8 and 8, 6 and 8.

IV. Study the same exercise as the last one with two figures and one. Thus, seeing 27 and 6, pass from 27 up to the next number that ends with 6 (or 36), catch the 9 through which you have to pass, and allow yourself to repeat as much as "27 and 9 are 36." Thus, the row of figures 17729638109 will give the following practice: 17 and 0 are 17; 77 and 5 are 82; 72 and 7 are 79; 29 and 7 are 36; 96 and 7 are 103; 63 and 5 are 68; 38 and 3 are 41; 81 and 9 are 90; 10 and 9 are 19.

V. In a number of two figures, practise writing down the units at the moment that you are keeping the attention fixed upon the tens. In the preceding exercise, for instance, write down the results, repeating the tens with emphasis at the instant of writing down the units.

VI. Learn the multiplication-table so well as to name the product

the instant the factors are seen ; that is, until 8 and 7, or 7 and 8, suggest 56 at once, without the necessity of saying " 7 times 8 are 56." Thus looking along a row of numbers, as 39706548, learn to name the products of every successive pair of digits as fast as you can repeat them, namely, 27, 63, 0, 0, 30, 20, 32.

VII. Having thoroughly mastered the last exercise, learn further, on seeing three numbers, to augment the product of the first and second by the third without any repetition of words. Practise until 3, 8, 4, for instance, suggest 3 times 8 and 4, or 28, without the necessity of saying " 3 times 8 are 24, and 4 is 28." Thus, 179236408 will suggest the following practice, 16, 65, 21, 12, 22, 24, 8.

VIII. Now, carry the last still further, as follows : Seeing four figures, as 2, 7, 6, 9, catch up the product of the first and second, increased by the third, as in the last, without a helping word ; name the result, and add the next figure, name the whole result, laying emphasis upon the tens. Thus, 2, 7, 6, 9, must immediately suggest " 20 and 9 are 29." The row of figures 773698974 will give the instances 52 and 6 are 58 ; 27 and 9 are 36 ; 27 and 8 are 35 ; 62 and 9 are 71 ; 81 and 7 are 88 ; 79 and 4 are 83.

IX. Having four numbers, as 2, 4, 7, 9, vary the last exercise as follows : Catch the product of the first and second, increased by the third ; but instead of adding the fourth, go up to the next number that ends with the fourth, as in exercise IV. Thus, 2, 4, 7, 9, are to suggest " 15 and 4 are 19." And the row of figures 1723968929 will afford the instances 9 and 4 are 13 ; 17 and 2 are 19 ; 15 and 1 are 16 ; 33 and 5 are 38 ; 62 and 7 are 69 ; 57 and 5 are 62 ; 74 and 5 are 79.

X. Learn to find rapidly the number of times a digit is contained in given units and tens, with the remainder. Thus, seeing 8 and 53, arrive at and repeat " 6 and 5 over." Common short division is the best practice. Thus, in dividing 236410792 by 7,

$$7)236410792$$
$$33772970, \text{ remainder } 2.$$

All that is repeated should be 3 and 2 ; 3 and 5 ; 7 and 5 ; 7 and 2 ; 2 and 6 ; 9 and 4 ; 7 and 0 ; 0 and 2.

In performing the several rules, proceed as follows:

ADDITION. Not one word more than repeating the numbers written in the following process: the accented figure is the one to be written down; the doubly accented figure is carried (and don't *say* "carry 3," but do it).

47963
1598
26316
54792
819
6686
_____
138174

6, 15, 17, 23, 31, $3''4'$; 11, 12, 21, 22, 31, $3''7'$; 9, 17, 24, 27, 32, $4''1'$; 10, 14, 20, 21, $2''8'$; 7, 9, $1'3'$.

In verifying additions, instead of the usual way of omitting one line, adding without it, and then adding the line omitted, verify each column by adding it both upwards and downwards.

SUBTRACTION. The following process is enough. The carriages, being always of *one*, need not be mentioned.

From 79436258190
Take 58645962738
_____
20790295452

8 and $2'$, 4 and $5'$, 7 and $4'$, 3 and $5'$, 6 and $9'$, 10 and $2'$, 6 and $0'$, 4 and $9'$, 7 and $7'$, 9 and $0'$, 5 and $2'$. It is useless to stop and say, 8 and 2 make 10; for as soon as the 2 is obtained, there is no occasion to remember what it came from.

MULTIPLICATION. The following, put into words, is all that need be repeated in the multiplying part; the addition is then done as usual The unaccented figures are carried.

670383
9876
_____
4022298
4692681
5363064
6033447
_____
662070250 8

$18'$, $49'$, $22'$, $2'$, $42'$, $4'0'$,
$21'$, $58'$, $26'$, $2'$, $49'$, $4'6'$,
$24'$, $66'$, $30'$, $3'$, $56'$, $5'3'$,
$27'$, $74'$, $34'$, $3'$, $63'$, $6'0'$.

Verify each line of the multiplication and the final result by casting out the nines. (*Appendix* II. p. 166.)

It would be almost as easy, for a person who has well practised the 8th exercise, to add each line to the one before in the process, thus:

                670383
                 9876
                ────────
                4022298   8 ; 21 and 9 are 30′ ; 59 and 2
               50949108   are 61′ ; 27 and 2 are 29 ; 2
              587255508   and 2 are 4′ ; 49 and 0 are 49′ ;
             6620702508   46 and 4 are 5′0′.

On the right is all the process of forming the second line, which completes the multiplication by 76, as the third line completes that by 876, and the fourth line that by 9876.

DIVISION. Make each multiplication and the following subtraction in one step, by help of the process in the 9th exercise, as follows:

              27693)441972809662(15959730
                    165042
                    265778
                    165410
                    269459
                    202226
                     83756
                      6772

The number of words by which 26577 is obtained from 165402 (the multiplier being 5) is as follows : 15 and 7′ are 2″2 ; 47 and 7′ are 5″4 ; 35 and 5′ are 4″0 ; 39 and 6′ are 4″5 ; 14 and 2′ are 16. .

The processes for extracting the square root, and for the solution of equations (*Appendix* XI.), should be abbreviated in the same manner as the division.*

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* The teacher will find further remarks on this subject in the *Companion to the Almanac* for 1844, and in the *Supplement to the Penny Cyclopædia*, article *Computation*.